

Robot Navigation Under MITL Constraints Using Time-Dependent Vector Field Based Control

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Abstract—In this work, we consider the problem of robot navigation, under spatial and temporal constraints, modeled as Metric Interval Temporal Logic (MITL) formulas. We introduce appropriate control schemes, driven by time-dependent vector fields, that satisfy both the problems of (a) entering an arbitrary neighborhood of the workspace within a given time interval, and, (b) avoiding collision with any given obstacle. We model the problems (a) and (b) as MITL formulas, defined upon a specific class of atomic propositions, and proceed in building more complex MITL expressions that can be decomposed into a conjunction of the former formulas. Finally, we propose a way to generate a hybrid automaton, whose execution satisfies the given MITL formula, by appropriately composing the control schemes. We validate our methodology via a numerical simulation.

I. INTRODUCTION

Motion and task planning constitute a fundamental problem in robotics, and still remain an active research topic in many respects. Many efficient approaches have been proposed, ranging from standard artificial intelligence, and temporal logic planning methods [1], [2], to methods based on artificial potential functions [3], [4], [5].

However, complex planning objectives, incorporating spatial and temporal constraints, are becoming all the more essential in robot navigation. For this reason motion planning under timed temporal logic has been recently studied [6], [7], [8]. In [6], the navigation problem is formulated as a mixed-integer optimization problem while, in [7] the system is discretized and approximated by a complex timed automaton, which, in turn, is analyzed by model checking tools.

On the other hand, methods based on the closed-loop evaluation of vector fields [3], [4], [9], although popular amongst researchers – owing to their low complexity, and their ability to simultaneously tackle both the motion planning and control problems – they are not able to handle temporal constraints. However, the authors in [10] and [11] recast the aforementioned problem by requiring that a robot be driven to a predefined neighborhood of the desired configuration in predetermined time. They proposed a novel vector field that ensures obstacle avoidance and facilitates the use of the Prescribed Performance Control technique [12], [13] to impose predetermined convergence to the desired configuration, thus resulting in a time-varying vector field planner.

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In order to tackle timed tasks in real-time while avoiding the increased computational complexity of timed temporal logic, the authors in [14] propose the construction of a hybrid automaton [15] that enables the decoupling of the navigation problem and the task sequencing. The hybrid automaton consists of appropriate control schemes that change subject to certain events, created by the sequence generator.

In this work, we consider the problem of real-time robot navigation in sphere world configuration spaces, which can be extended to generalized sphere worlds, upon global knowledge of the environment. We introduce a time-dependent vector field function, which results in a prescribed performance control scheme as defined in [12], such that the robot is driven from any initial configuration to an arbitrary neighborhood of the workspace within a given time interval. Additionally, we borrow from [10], and define a vector field driven control scheme, such that, collision avoidance with any given obstacle is established. We introduce a class of atomic propositions to describe the above main problems as MITL formulas, and proceed in building more complex MITL expressions that can be decomposed into these. Finally, we propose a way to generate a hybrid automaton, whose execution satisfies the given MITL formula, by appropriately composing the control schemes associated with the two main formulas. The proposed methodology is guaranteed to satisfy the MITL specifications, and is validated by a non-trivial numerical simulation.

II. PRELIMINARIES

A. Workspace and Robot Kinematics

We consider a point robot¹ operating in a bounded workspace $\mathcal{W} \subset \mathbb{R}^n$ with $n \in \mathbb{N}_{\geq 2}$ and denote its position by $x \in \mathcal{W}$. The workspace is assumed to be an open ball centered at the origin

$$\mathcal{W} \triangleq \{q \in \mathbb{R}^n : \|q\| < r_{\mathcal{W}}\} \quad (1)$$

where $r_{\mathcal{W}} \in \mathbb{R}_{>0}$ is the workspace radius. The workspace can be populated with $m \in \mathbb{N}$ closed sets O_i , $i \in \mathcal{J} \triangleq \mathbb{N}_{\leq m}$, corresponding to obstacles. In particular, each obstacle $i \in \mathcal{J}$ is a ball centered at p_i with radius $r_i \in \mathbb{R}_{>0}$,

$$O_i \triangleq \{q \in \mathcal{W} : \|q - p_i\| \leq r_i\}, \quad \forall i \in \mathcal{J}. \quad (2)$$

Assumption 1. The obstacles are assumed to be static, i.e. p_i and r_i do not depend on time, and the free space $\mathcal{F} \triangleq \mathcal{W} \setminus \bigcup_{i \in \mathcal{J}} O_i$ is assumed to be a sphere world, i.e., each

¹Treating a robot with volume can be achieved by “transferring” its volume to the other workspace entities and considering it as a point.

obstacle O_i is contained in workspace $\mathcal{W}$ and the obstacle sets are pairwise disjoint. This assumption will simplify further analysis, but can be alleviated as shown in [9], [11].

Assumption 2 (Single Integrator Kinematics). *The robot kinematics are assumed to be given by the first order holonomic kinematic model*

$$\dot{x} = u \quad (3)$$

where $x(0) = x_0 \in \mathcal{W}$ and $u \in U \subseteq \mathbb{R}^n$.

B. Metric Interval Temporal Logic (MITL)

Definition 1. An atomic proposition $\pi : \mathcal{W} \rightarrow \{\top, \perp\}$ is a statement about the system variables (x) that takes the Boolean constant **True**($\top$) or **False**($\perp$) for some given values of the state variables.

Definition 2. The syntax of MITL formulas are defined according to the following grammar rules:

$$\phi ::= \top \mid \pi \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 U_I \phi_2,$$

where $I \subseteq [0, \infty]$ is an interval with end points in $\mathbb{Q}_{\geq 0} \cup \{\infty\}$, π is an atomic proposition, and $\top$ and $\perp (= \neg\top)$ are the Boolean constants true and false, respectively.

Definition 3. The semantics of an MITL formula ϕ is recursively defined over a trajectory x_t as:

$$\begin{aligned} x_t &\models \pi \text{ iff } \pi(x_t) = \top \\ x_t &\models \neg\pi \text{ iff } \pi(x_t) = \perp \\ x_t &\models \phi_1 \wedge \phi_2 \text{ iff } x_t \models \phi_1 \text{ and } x_t \models \phi_2 \\ x_t &\models \phi_1 U_I \phi_2 \text{ iff } \exists s \in I \text{ s.t. } x_{t+s} \models \phi_2 \text{ and } \forall s' \leq s, x_{t+s'} \models \phi_1. \end{aligned}$$

Other Boolean operators can also be expressed [16], and the following MITL operators of special interest can be defined:

- $\Diamond_I \phi \equiv \top U_I \phi$ meaning that ϕ will eventually become true within the time interval I , and
- $\Box_I \phi \equiv \neg \Diamond_I \neg \phi$ meaning that ϕ is always true for the time interval I .

C. Hybrid Automaton

A Hybrid Automaton is a dynamical system that describes the evolution in time of the values of a set of discrete and continuous variables [14], [15].

Definition 4 (Hybrid Automaton). A hybrid automaton H is an eleven tuple $H = (Q, X, E, U, f, \delta, \text{Inv}, \text{guard}, \rho, q_0, x_0)$, where

- Q is a set of discrete states or modes;
- X is a set of continuous state space (normally $X = \mathbb{R}^n$);
- E is a finite set of events;
- U is a set of admissible controls (normally $U \subseteq \mathbb{R}^m$);
- $f : Q \times X \times U \rightarrow X$ is a vector field;
- $\delta : Q \times X \times E \rightarrow Q$ is a discrete state transition function;
- $\text{Inv} \subseteq Q \times X$ is a set defining an invariant condition (also called domain);
- $\text{guard} \subseteq Q \times Q \times X$ is a set defining a guard condition;
- $\rho : Q \times Q \times X \times E \rightarrow X$ is a reset function;
- q_0 is an initial discrete state;
- x_0 is an initial continuous state.

III. SAFE NAVIGATION IN PRESCRIBED TIME

In this section, we deal with the problem of navigation under temporal and spatial constraints, and in particular with the two main specifications met in the problem, i.e. within a given time interval, (a) enter a predefined neighborhood of the workspace, and (b) avoid any obstacles. We formulate (a) and (b) as MITL expressions, by introducing a set $\mathcal{P}$ of propositions that describe the presence of the robot in a certain area of the workspace:

Definition 5. The set $\mathcal{P}$ is a set of atomic propositions p , each described by a pair of parameters $(x_p, r_p) \in \mathcal{W} \times \mathbb{R}_{\geq 0}$, which describe the presence of the robot in a predefined neighborhood of a point $x_p \in \mathcal{W}$, i.e.,

$$\begin{aligned} &\text{if } p \in \mathcal{P} \text{ then} \\ &p(x) = \top \Leftrightarrow \|x - x_p\| \leq r_p. \end{aligned}$$

In what follows, we propose control laws, driven by appropriately defined time-dependent vector fields, such that for $p \in \mathcal{P}$, the MITL expressions $\phi_a = \Diamond_J p$, which states that “eventually within a time interval J enter an area of $\mathcal{W}$ ”, and $\phi_b = \Box_I \neg p$, which states that “always on the time interval I avoid an area of $\mathcal{W}$ ”, are satisfied.

A. Navigation within given time interval

Consider the satisfiability problem:

Problem 1 (Navigation within a given time interval). Let $x(t) \in \mathcal{W}$, $t \in [0, \tau_p]$ be a trajectory satisfying the kinematics (3). Then the problem of navigating from $x(0)$ to a predefined neighborhood of $\mathcal{W}$ within a given time interval $J = [0, \tau_p]$, can be formulated as:

$$\begin{aligned} &x_0 \models \Diamond_J p \\ &\text{s.t. } p = p_{(x_p, r_p)} \in \mathcal{P} \\ &J = [0, \tau_p] \\ &x_0 \triangleq x(0) \in \mathcal{W}, \end{aligned}$$

which states that $\|x(t) - x_p\| \leq r_p$ for some $t \in [0, \tau_p]$.

Consider also the following problem studied in [10]:

Problem 2 (Prescribed Time Scale Navigation Problem). Assuming single integrator robot kinematics,

$$\dot{x} = u$$

and for any pair of initial and final configurations $(x_0, x_d) \in \mathcal{W}^2$, and any pair (r, τ) with $r, \tau > 0$, determine a time-varying controller $u : \mathbb{R}_{\geq 0} \times \mathcal{W} \rightarrow \mathbb{R}^n$ such that the workspace space $\mathcal{W}$ is forward invariant and

$$\|x(t) - x_d\| < r, \quad \forall t \geq \tau. \quad (4)$$

Intuitively, equation (4) means that by time τ the robot will have entered a ball of radius r centered at the desired configuration, and remain inside it thereafter.

Then, unraveling the definitions, we have that:

Proposition 1. A solution u , of Problem 2 yields a closed-loop trajectory $x(\cdot)$, such that Problem 1 is satisfied for $(x_p, r_p, \tau_p) = (x_d, r, \tau)$.

As discussed in [10], we adopt the notion of prescribed performance control technique [12], [13], such that

$$\gamma(x) = \|x - x_d\|^2 < \rho(t), \quad \forall t \in \mathbb{R}_{\geq 0} \quad (5)$$

where ρ is a designer-specified, smooth, bounded and decreasing function of time, here

$$\rho(t) = (\rho_0 - \rho_\infty)e^{-lt} + \rho_\infty$$

satisfying $\rho_0 > \gamma(x_0)$, $\rho_\infty < r^2$, $l = l(r, \tau) \geq -\frac{1}{\tau} \ln\left(\frac{r^2 - \rho_\infty}{\rho_0 - \rho_\infty}\right)$.

The proposed controller is then defined as

$$u_\gamma(t, x) = -\left(k\varepsilon(\xi) + \frac{1}{2}\alpha(t)\right)(x - x_d), \quad k > 0 \quad (6)$$

where $\varepsilon(\xi) \triangleq T(\xi) = \ln\left(\frac{1}{1-\xi}\right)$, $\xi \in \mathbb{R}_{<1}$, and $\alpha(t) \triangleq -\frac{\dot{\rho}(t)}{\rho(t)} > 0$, and can be expressed as $u_\gamma = -\nabla U_\gamma$ where

$$U_\gamma(t, x) = \frac{1}{2} \left[\left(k + \frac{\alpha(t)}{2}\right) \gamma(x) + k\varepsilon(t, x)(\gamma(x) - \rho(t)) \right], \quad k > 0 \quad (7)$$

Proposition 2. *The control law u_γ defined in (6) is a solution to Problem 2.*

Proof. The detailed proof is given in [10]. $\square$

B. Obstacle Avoidance

Consider the satisfiability problem:

Problem 3 (Obstacle Avoidance). *Let $x(t) \in \mathcal{W}$, $t \in [0, \tau_p]$ be a trajectory satisfying the kinematics (3). Then the problem of not entering a predefined neighborhood of $\mathcal{W}$ throughout a given time interval $J = [0, \tau_p]$, can be formulated as:*

$$\begin{aligned} x_0 &\models \Box_{I \neg p}, \\ \text{s.t. } p &= p_{(x_p, r_p)} \in \mathcal{P} \\ J &= [0, \tau_p] \\ x_0 &\triangleq x(0) \in \mathcal{W} \setminus \{q \in \mathcal{W} : \|q - x_p\| \leq r_p\} \end{aligned}$$

which states that $\|x(t) - x_p\| > r_p$ for all $t \in [0, \tau_p]$.

Consider also the following problem:

Problem 4 (Obstacle Avoidance). *Assuming single integrator robot kinematics,*

$$\dot{x} = u$$

and for any obstacle $O_i \triangleq \{q \in \mathcal{W} : \|q - p_i\| \leq r_i\}$, $i \in \mathcal{J}$, any initial configuration $x_0 \in \mathcal{F}_i \triangleq \mathcal{W} \setminus O_i$, and any $\tau > 0$, determine a time-varying controller $u : \mathbb{R}_{\geq 0} \times \mathcal{F}_i \rightarrow \mathbb{R}^n$ such that the free space $\mathcal{F}_i$ is forward invariant.

Intuitively, $\mathcal{F}_i$ being forward invariant means that for every $t \in [0, \tau]$ the robot will avoid obstacle O_i .

Proposition 3. *A solution u , of Problem 4 yields a closed-loop trajectory $x(\cdot)$, such that Problem 3 is satisfied for $(x_p, r_p, \tau_p) = (p_i, r_i, \tau)$.*

We adopt the vector field $\beta_i(x) = \inf\{\|q - p_i\|^2 : q \in \mathcal{S}(x)\}$, $i \in \mathcal{J}$, introduced in [10], where $\mathcal{S}(x) \triangleq \{q \in \mathbb{R}^n : q = (1 - \lambda)x + \lambda x_d, \lambda \in [0, 1]\} \subset \mathcal{W}$, with

$x_d \in \mathcal{F}_i$ being a desired configuration, to define the feedback law

$$u_{\beta_i}(x) \triangleq \frac{\sigma_{i,\delta}(x)}{d_i(x)} \left(\nabla \beta_i(x) + \mathbb{1}_{\mathcal{M}_i}(x) (x - x_d)^\perp \right) \quad (8)$$

where $d_i(x) = \|x - p_i\|^2 - r_i^2$, $\sigma_{i,\delta}(x) : \mathbb{R} \rightarrow [0, 1]$ are C^1 switches that make the effect of the obstacle O_i local, as defined in [10], and the term $\mathbb{1}_{\mathcal{M}_i}(x - x_d)^\perp$, where $(x - x_d)^\perp$ denotes a vector normal to $x - x_d$, and $\mathbb{1}_{\mathcal{M}_i}$ denotes the indicator function of the set $\mathcal{M}_i \triangleq \{q \in \mathcal{W} \setminus \{x_d\} : q = p_i + \mu(p_i - x_d), \mu \in \mathbb{R}_{\geq 0}\}$, introduces a discontinuity necessary to prevent the robot from remaining in the set $\mathcal{M}_i$.

Proposition 4. *The control law u_{β_i} defined in (8) is a solution to Problem 4.*

Proof. The detailed proof is given in [10]. $\square$

C. Composition of the proposed controllers

In this section we simultaneously investigate the problems of navigation and obstacle avoidance in prescribed time, by appropriately composing the controllers defined in (6), and (8).

Consider the satisfiability problem:

Problem 5 (Safe navigation in prescribed time). *Let $x(t) \in \mathcal{W}$, $t \in [0, t_f]$ be a trajectory satisfying the kinematics (3), and $O_i \triangleq \{q \in \mathcal{W} : \|q - p_i\| \leq r_i\}$, $\forall i \in \mathcal{J}$, represent the obstacles of the workspace $\mathcal{W}$, as defined in (2). Then the problem of safe navigation in prescribed time can be formulated as:*

$$\begin{aligned} x_0 &\models (\Diamond_I p) \wedge \left(\bigwedge_{i \in \mathcal{J}} \Box_{I \neg p_i} \right) \\ \text{s.t. } p &= p_{(x_p, r_p)} \in \mathcal{P} \\ p_i &= p_{(x_i, r_i)} \in \mathcal{P}, \quad i \in \mathcal{J} \\ J &= [0, \tau], \quad I = [0, t_f], \quad \tau \leq t_f \\ x_0 &\triangleq x(0) \in \mathcal{W} \setminus \bigcup_{i \in \mathcal{J}} O_i \end{aligned}$$

Then it follows from the work of Vrohidis et. al in [10] that, given the controllers u_γ and u_{β_i} , $i \in \mathcal{J}$ as defined in (6), and (8), the following proposition holds

Proposition 5. *The controller*

$$u(t, x) \triangleq u_\gamma(t, x) + u_{\beta_i}(x) \quad (9)$$

where

$$u_{\beta_i}(x) \triangleq \sum_{i \in \mathcal{J}} u_{\beta_i}(x) \quad (10)$$

along with system (3), yields $x(t)$, $t \in I$ that satisfies Problem 5, for $(x_p, r_p) = (x_d, r)$, and $(x_i, r_i) = (p_i, r_i)$, $\forall i \in \mathcal{J}$, as long as the obstacle sets are pairwise disjoint, i.e., $O_i \cap O_j = \emptyset$, for all $i, j \in \mathcal{J}$, $i \neq j$, and $\|p_i - p_j\| > r_i + r_j + 2\bar{r}$, $\forall i, j \in \mathcal{J}$, $i \neq j$.

IV. COMPLEX MITL EXPRESSIONS IN ROBOT NAVIGATION

In this section, we investigate more complex MITL expressions that are useful in robot navigation, and can be decomposed into a conjunction of the simpler formulas $\phi_a = \Diamond_J p$ and $\phi_b = \Box_I \neg p$ as defined in Section III. Using the controllers u_γ and u_β , $i \in \mathcal{J}$ defined in (6), and (8), we define time-varying control schemes, modeled as hybrid automata (II-C), such that the original MITL expressions are satisfied.

A. Task Execution

Because of our adherence to the set of atomic propositions $\mathcal{P}$ (Definition 5), some MITL expressions become indifferent in robot navigation applications. For example, given the robot kinematics (3), the formula $\Box_I p = \top$, $p \in \mathcal{P}$, is trivially satisfied by $u \equiv 0$ if $x(0) \models p$, or not at all if $x(0) \not\models p$.

In this regard, it would be useful to request that the robot, once it is close to a desired configuration, execute a specific task, e.g. move in a certain way or grasp an object. We describe the execution of a task as an atomic proposition q belonging to the following set $\mathcal{Q}$:

Definition 6. The set $\mathcal{Q}$ is a set of atomic propositions q , each described by a pair of parameters (x_q, r_q) , and a function $f_q : \mathbb{R}_{\geq 0} \times \mathcal{C}^n[0, \infty) \times \mathcal{W} \times \mathbb{R}_{> 0} \rightarrow \mathcal{W}$ which describe the presence of the robot in a predefined neighborhood of a point $x_q \in \mathcal{W}$, and the execution of a given task, respectively, i.e.,

$$\text{if } q \in \mathcal{Q} \text{ then} \\ q(x) = \top \Leftrightarrow \begin{cases} \|x - x_q\| \leq r_q \\ f_q(t, x(\cdot), x_q, r_q) = 0 \end{cases}$$

where t denotes time, x denotes the position of the robot at the given time, and $x(\cdot) \in \mathcal{C}^n(\mathbb{R})$ denotes the trajectory of the robot as a function of time.

Assumption 3. We assume that every atomic proposition $q \in \mathcal{Q}$ used, is associated with a time interval $I = [0, t_f]$ and a controller $u_\psi(\cdot)$ (that may depend on the function $x(\cdot)$), such that

$$\dot{x}(t) = u_\psi(t) \Leftrightarrow \begin{cases} \|x(t) - x_q\| \leq r_q \\ f_q(t, x, x_q, r_q) = 0 \end{cases}, t \in I$$

i.e., $\phi_q = \Box_I q$ is satisfied, provided that $\|x(0) - x_q\| \leq r_q$. In other words, we request that the task can be executed by a closed loop system defined by the designer of the motion planning specifications, and ensures that $\|x - x_q\| \leq r_q$ will always hold in I .

Therefore, in view of Definition 6, the expression

$$\Diamond_J \Box_I q, q \in \mathcal{Q}$$

now takes the meaning that within time interval $J = [0, t_J]$, the robot will have entered an area of $x_q \in \mathcal{W}$, and from that time, say $\tau \in J$, and until $\tau + |I|$, where $|I| = \int_{I \in I} dt$ is the duration of I , the robot will be executing the task $f_q(t, x, x_q, r_q) = 0$. Noting that $\mathcal{Q}$ reduces to $\mathcal{P}$ for the trivial

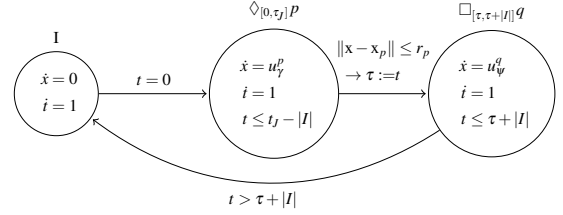


Fig. 1. A hybrid automaton satisfying the expression $\phi_{ea} = \Diamond_J \Box_I q$, $q \in \mathcal{Q}$, $J = [0, t_J]$. Note that ϕ_{ea} can, equivalently, be written as $\phi_{ea} = \Diamond_{[0, t_J]} p \wedge \Box_{[\tau, \tau + |I|]} q$ where $\tau_J = t_J - |I|$, $\tau = \min \{t \in [0, \tau_J] : x(t) \models p\}$.

choice of $f_q(t, x, x_s, r_q) \equiv 0$, i.e. when there is no additional task to be executed, we get the following useful relation:

$$\Diamond_J \Box_I q \equiv \Diamond_{[0, t_J]} p \wedge \Box_{[\tau, \tau + |I|]} q \quad (11)$$

where $\tau_J = t_J - |I|$, $\tau = \min \{t \in [0, \tau_J] : x(t) \models p\}$, and $p \in \mathcal{P}$ and $q \in \mathcal{Q}$ are associated by $\{x_q, r_q\} = \{x_p, r_p\}$.

It follows from (11) and Assumption 3 that

Proposition 6. The expression

$$\phi_{ea} = \Diamond_J \Box_I q, q \in \mathcal{Q}, J = [0, t_J]$$

is satisfied by $x(0)$, if $\dot{x} = u_{ea}$, with

$$u_{ea} = \begin{cases} u_\gamma^p(t, x), & t \in [0, \tau] \\ u_\psi^q(t), & t \in [\tau, \tau + |I|] \end{cases} \quad (12)$$

where $\tau = \min \{t \in [0, \tau_J] : x(t) \models p\}$, $\tau_J = t_J - |I|$, u_γ^p is defined as in (6) for the proposition p which is associated with q by $\{x_q, r_q\} = \{x_p, r_p\}$, and u_ψ^q is designed for proposition q and is known (Assumption 3).

We can use the controller defined in (12) to construct a hybrid automaton that satisfies the formula $\phi_{ea} = \Diamond_J \Box_I q$, $q \in \mathcal{Q}$, $J = [0, t_J]$. Because of space limitation, we omit the formal definition of the hybrid automaton and proceed with the usual graphical representation in Figure 1.

Remark 1. We note that the time instance τ in (11) and (12) is not known a priori, but results from the execution of the system (3) with $u = u_{ea}$. In fact, time τ essentially corresponds to the event $x(t) \models p$ triggering an appropriately defined state transition function of the hybrid automaton, as shown in Fig. 1. As a result, such time τ may not exist at all, in which case, the problem is not satisfiable (see Section IV-D).

B. Precedence Constraints

It is easy to see that, by definition:

$$\phi_1 \mathbf{U}_I \phi_2 \equiv \Box_{[0, \tau]} \phi_1 \wedge \Diamond_{[0, t_f]} \phi_2$$

where $I = [0, t_f]$, ϕ_1, ϕ_2 are MITL formulas, and $\tau \in I$ is such that $\tau = \min \{t \in I : x(t) \models \phi_2\}$.

An important use of the timed Until operator in motion planning is to impose *precedence constraints*, i.e., a proposition p_1 be satisfied prior to another, p_2 , which is captured in the MITL expression:

$$\neg p_2 \mathbf{U}_I p_1 \wedge \Diamond_J p_2 \equiv \Diamond_I p_1 \wedge \Box_{[0, \tau]} \neg p_2 \wedge \Diamond_{[\tau, t_{Jf}]} p_2 \quad (13)$$

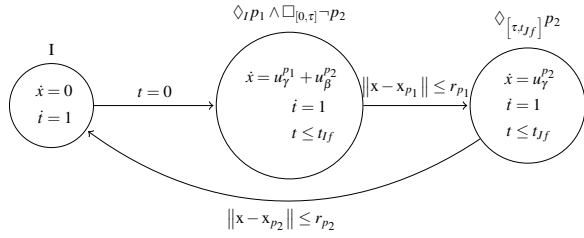


Fig. 2. A hybrid automaton satisfying the expression $\phi_{pr} = \neg p_2 \mathbf{U}_I p_1 \wedge \Diamond_J p_2$, $p_1, p_2 \in \mathcal{P}$, where $I = [0, t_{Jf}]$, $J = [0, t_{Jf}]$. Note that ϕ_{pr} can, equivalently, be written as $\phi_{pr} = \Diamond_I p_1 \wedge \square_{[0, \tau]} \neg p_2 \wedge \Diamond_{[\tau, t_{Jf}]} p_2$ where $\tau = \min \{t \in I : x(t) \models p_1\}$.

where $I = [0, t_{Jf}]$, $J = [0, t_{Jf}]$, $t_{Jf} > t_{If}$, $p_1, p_2 \in \mathcal{P}$, and $\tau = \min \{t \in I : x(t) \models p_1\}$ (see Remark 1).

It follows from eq. (13) and Proposition 5 that

Proposition 7 (Precedence Constraints). *The expression*

$$\neg p_2 \mathbf{U}_I p_1 \wedge \Diamond_J p_2, \quad p_1, p_2 \in \mathcal{P}$$

where $I = [0, t_{Jf}]$, and $J = [0, t_{Jf}]$ is satisfied by $x(0)$, if $\dot{x} = u_{pr}$, with

$$u_{pr} = \begin{cases} u_{\gamma}^{p_1}(t, x) + u_{\beta}^{p_2}(x), & t \in [0, \tau_1] \\ u_{\gamma}^{p_2}(t, x), & t \in [\tau_1, \tau_2] \end{cases} \quad (14)$$

where $\tau_1 = \min \{t \in I : x(t) \models p_1\}$, and $\tau_2 = \min \{t \in [\tau_1, t_{Jf}] : x(t) \models p_2\}$ (see Remark 1), $u_{\gamma}^{p_1}$, $u_{\gamma}^{p_2}$ are defined as in (6) for propositions p_1 and p_2 , respectively, and, $u_{\beta}^{p_2}$ is defined as in (10) for $\mathcal{J} = \{1\}$, and an obstacle O_1 defined by the parameters of proposition p_2 .

We can use the controller defined in (14) to construct a hybrid automaton that satisfies the formula $\phi_{pr} = \neg p_2 \mathbf{U}_I p_1 \wedge \Diamond_J p_2$, $p_1, p_2 \in \mathcal{P}$. Due to space limitation, we only provide the graphical representation in Figure 2.

C. A complex example

Consider the following motion planning specification:

“Until time t_u , enter the sphere $\{x \in \mathcal{W} : \|x - x_a\| \leq r_a\}$, and, for time duration of $t_a < t_u$, do the task $f_a(t, x, x_a, r_a) = 0$. After this is finished ($t \leq t_u$), enter the sphere $\{x \in \mathcal{W} : \|x - x_b\| \leq r_b\}$ before time $t_e > t_u$, and, for time duration of t_b , do the task $f_b(t, x, x_b, r_b) = 0$. Meanwhile, at all times, avoid the obstacles $\{x \in \mathcal{W} : \|x - p_i\| \leq r_i\}$, $i \in \mathcal{J}$.”

This is captured by the following MITL formula:

$$\phi = (\Diamond_{[0, t_e + t_b]} \square_{[0, t_b]} B^q) \wedge (\neg B^q \mathbf{U}_{[0, t_u]} \square_{[0, t_a]} A^q) \wedge \left(\bigwedge_{i \in \mathcal{J}} \square \neg O_i \right) \quad (15)$$

where $t_a < t_u < t_e$, $A^q, B^q \in \mathcal{Q}$ are associated with $A^p, B^p \in \mathcal{P}$ via $(x_p^A, r_p^A) = (x_q^A, r_q^A)$, and $(x_p^B, r_p^B) = (x_q^B, r_q^B)$, and $O_i \in \mathcal{P}$, $i \in \mathcal{J}$.

Following the same methodology we can write:

$$\phi = \Diamond_{[0, t_u - t_a]} A^p \wedge \square_{[\tau_1, \tau_1 + t_a]} A^q \wedge \square_{[0, \tau_1 + t_a]} \neg B^p \wedge \Diamond_{[\tau_1 + t_a, t_e]} B^p \wedge \square_{[\tau_2, \tau_2 + t_b]} B^q \wedge \left(\bigwedge_{i \in \mathcal{J}} \square \neg O_i \right)$$

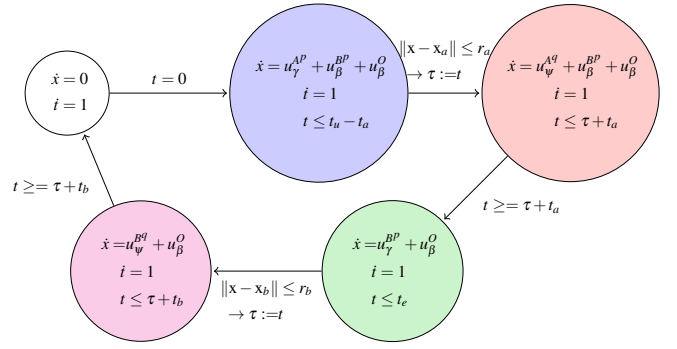


Fig. 3. A hybrid automaton satisfying the expression $\phi = (\Diamond_{[0, t_e + t_b]} \square_{[0, t_b]} B^s) \wedge (\neg B^s \mathbf{U}_{[0, t_u]} \square_{[0, t_a]} A^s) \wedge (\bigwedge_{i \in \mathcal{J}} \square \neg O_i)$.

where $t_a < t_u < t_e$, $\tau_1 = \min \{t \in [0, t_u - t_a] : x(t) \models A^p\}$, and $\tau_2 = \min \{t \in [\tau_1 + t_a, t_e] : x(t) \models B^p\}$ (see Remark 1).

Finally we can construct a hybrid automaton that satisfies the formula ϕ . Due to space limitation, we provide only the graphical representation in Figure 3.

D. Limitations

At first, we note that we are not addressing a decision, or optimization, problem, but instead propose a control scheme that satisfies a given MITL formula. As a result, formulas such as $\phi_1 = \Diamond_I (p_1 \vee p_2)$, $p_1, p_2 \in \mathcal{P}$, are not of particular interest and cannot be handled by the given approach. We assume that this has been taken care of by the designer of the MITL specifications.

Secondly, we stress that, as discussed in [14], our methodology reduces the time-constrained navigation problem into a scheduling problem for the generated hybrid automaton. We generate the hybrid automaton by decomposing the given MITL expression ϕ , into $\phi = \phi_1 \wedge \dots \wedge \phi_k$ such that the controllers u_i that make the system (3) satisfy ϕ are known for each ϕ_i , $i = 1, \dots, k$. In order to do that, we address the scheduling problem, using the auxiliary variables $\tau_i \in \{t : x(t) \models \phi_i\}$, $i = 1, \dots, k$, to define ϕ_j , $j \neq i$, as, for example, in (11) and (12). It follows from the above and from Remark 1, that the time instances τ_i , $i = 1, \dots, k$ are not uniquely defined and may not exist. Therefore, the task sequencing problem can be further addressed on its own, especially under constraints on the robot dynamics, or the control input.

V. SIMULATION RESULTS

To demonstrate the applicability of the proposed methodology, we simulate the hybrid automaton generated for the MITL formula (15), as shown in Figure 3.

The workspace is defined as the open ball $\mathcal{W} \triangleq \{q \in \mathbb{R}^2 : \|q\| < 15\}$. The parameter values are chosen such that $t_a = t_b = 3$, $t_u = 10$, $t_e = 22$, $(x_p^A, r_p^A) = (x_q^A, r_q^A) = ([0, 0], 1)$, and $(x_p^B, r_p^B) = (x_q^B, r_q^B) = ([3, 4], 2)$. The parameters (p_i, r_i) of the obstacles O_i , $i = 1, \dots, 10$ are given by $([0, 5], 3)$, $([6, 3], 2)$, $([-6, 3], 2)$, $([0, -7], 3)$, $([6, -5], 2)$, $([-6, -5], 2)$, $([10, 0], 2)$, $([-10, 0], 2)$, $([3, -0.9], 2)$, and $([-3, -0.9], 2)$, respectively. The task $f_a(t, x, x_a, r_a) = 0$ is defined such that the robot moves in a clockwise circular trajectory in a

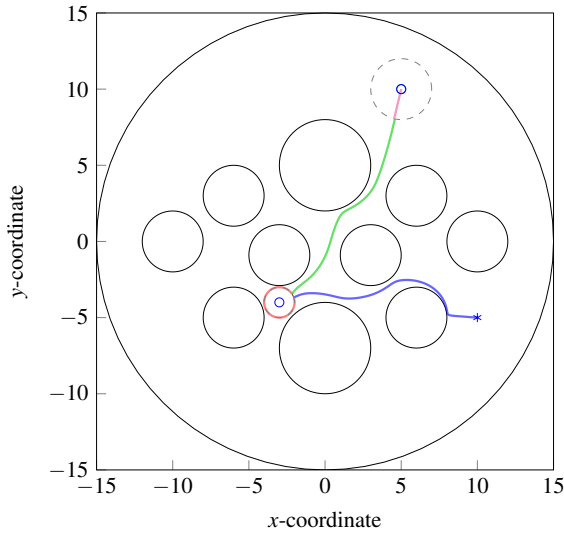


Fig. 4. Workspace and continuous solution of the hybrid automaton. Black circles correspond to the boundary of the obstacles. The asterisk indicates the initial configuration, and the blue circles correspond to the points x_q^A and x_q^B . The coloring of the trajectory is in correspondence with the coloring of the states of the hybrid automaton (Fig. 3).

constant distance $d_a \leq r_a = r_q^A$ around $x_a = x_q^A$ with a constant speed of 2. This is achieved by the closed-loop control law

$$u_{\psi}^{Aq}(x) = 2 \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \frac{x - x_a}{\|x - x_a\|},$$

where $d_a = \|x(\tau) - x_a\|$ is the distance from x_a calculated at time τ when the controller first gets activated.

The task $f_b(t, x, x_b, r_b) = 0$ is defined such that the robot converges to $x_b = x_q^B$ in finite time. This is achieved by the closed-loop control law

$$u_{\psi}^{Bq}(x) = -k_b \|x - x_b\|^{-\frac{1}{2}} (x - x_b),$$

for $k_b = \frac{2}{3}\sqrt{2}$. The robot kinematics are given by (3), and the controllers u_{γ} and u_{β} are as defined in (6), and (10). The parameter δ of (10) was set equal to 10. The parameters of controller u_{γ} are provided in Figure 5. The trajectory of the closed-loop hybrid system is depicted in Figure 4. The coloring of the trajectory is in correspondence with the coloring of the states of the hybrid automaton (see Figure 3 for comparison). Finally, Figure 5 illustrates the satisfaction of the temporal specifications described by the associated MITL formula.

VI. CONCLUSION

We have considered the problem of robot navigation, under spatial and temporal constraints, modeled as MITL formulas. We introduced appropriate control schemes, driven by time-dependent vector fields, and proposed a way to generate hybrid automata, whose execution satisfies the given MITL formulas, by appropriately composing the control schemes. Our methodology was validated via a complex numerical simulation. Future research efforts will be focused on the limitations of this work discussed in Section IV-D, and in particular on handling constraints in control input and robot kinematics.

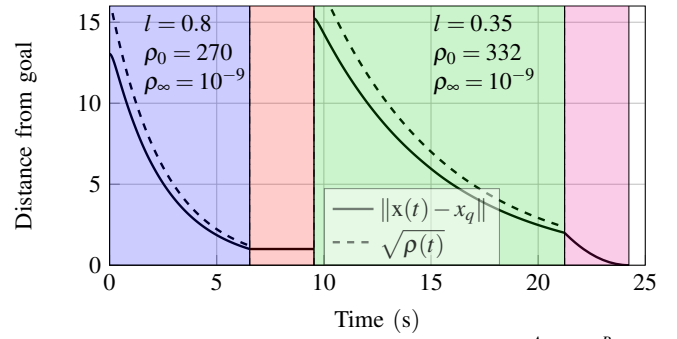


Fig. 5. Temporal evolution of the distance from points x_q^A and x_q^B . The coloring of the trajectory is in correspondence with the coloring of the states of the hybrid automaton (Fig. 3).

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